

Feedback Control of Turbulent Shear Flows by Genetic Programming

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Abstract

Turbulent shear flows have triggered fundamental research in nonlinear dynamics, like transition scenarios, pattern formation and dynamical modeling. In particular, the control of nonlinear dynamics is subject of research since decades. In this publication, actuated turbulent shear flows serve as test-bed for a nonlinear feedback control strategy which can optimize an arbitrary cost function in an automatic self-learning manner. This is facilitated by genetic programming providing an analytically treatable control law. Unlike control based on PID laws or neural networks, no structure of the control law needs to be specified in advance. The strategy is first applied to low-dimensional dynamical systems featuring aspects of turbulence and for which linear control methods fail. This includes stabilizing an unstable fixed point of a nonlinearly coupled oscillator model and maximizing mixing, i.e. the Lyapunov exponent, for forced Lorenz equations. For the first time, we demonstrate the applicability of genetic programming control to four shear flow experiments with strong nonlinearities and intrinsically noisy measurements. These experiments comprise mixing enhancement in a turbulent shear layer, the reduction of the recirculation zone behind a backward facing step, and the optimized reattachment of separating boundary layers. Genetic programming control has outperformed tested optimized state-of-the-art control and has even found novel actuation mechanisms.

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I. INTRODUCTION

The control of turbulence is of fundamental importance for understanding of animal motion and for many engineering applications. It serves as one of the most complex paradigms for methods of the control of dynamical systems in general. In fluid dynamics, many engineering applications benefit from feedback control, like drag reduction of transport vehicles or green energy harvesting of wind and water flows. In these fields, even small gain in energy savings are very important due to the huge number of vehicles, turbines, etc. involved. Apart from fluids, a generic control strategy for complex nonlinear systems is desired in life sciences, medicine, and a vast number of engineering applications.

Apart from the importance of turbulence for applications, it serves as a paradigm for the study of complex dynamical systems. Fluid dynamics have led to many important discoveries and poses challenges to physics theory. Pattern formation, chaos and bifurcations, would not have been found and studied without the driving force of turbulent flows. So, we follow this line of thought and use turbulent flows as prototypic example for a complex dynamical system. Yet, control strategies are best developed for simple nonlinear dynamical systems. Consequently, we study in this publication the control of two systems with nonlinear features of turbulence: (1) two coupled forced oscillators which are linearly uncontrollable, and (2) a forced Lorenz system, where we optimize for mixing, i.e. aim at large Lyapunov exponents. Based on these results, we apply our control algorithm to experiments with turbulence.

For many laminar flows, linear control theory is used for the stabilization of a steady solution by a local linearization of the Navier-Stokes equation. Corresponding numerical and experimental stabilization studies include virtually any configuration, e.g. wakes [1], cavity flows [2–4], flows of backward-facing step [5], boundary-layer flows [6].

Turbulent flows, however are next to impossible to stabilize, because of the huge number of degrees of freedom, the non-normal amplification mechanisms, complex bifurcations and nonlinear frequency cross talk. In addition, real-world constraints complicate control, e.g. the cost of energy of actuators, or their blank inability to control to a steady solution, because actuators cannot provide enough energy or do not have the authority to force the fluid into a steady flow. Linear control fails to handle frequency cross-talk between the coherent structures, the mean flow and the stochastic small-scale fluctuations. Yet, frequency cross-talk is an important actuation opportunity as demonstrated by successful

wake stabilization with high-frequency actuation [7–9] and low-frequency forcing [10]. For real-world applications, model-based control of an experiment requires a robust control-oriented reduced-order model which is still a large challenge at this moment. Such a reduced-order model needs — at minimum — to resolve the uncontrolled and controlled turbulent coherent structures including the transients between them.

Intriguingly, animals have found feedback flow control solutions neither knowing the Navier-Stokes equation nor reduced models thereof. Examples are eagles whose feathers near the trailing edge move up to prevent stall. Or dolphins whose compliant skins delay transition and thus decrease drag. More than 50 years ago, Rechenberg [11] and Schwefel [12] have imitated nature’s evolutionary principles to solve engineering flow problems. Subsequently, evolutionary algorithms have rapidly evolved culminating in the genetic algorithm [13] and genetic programming [14] and powerful methods of machine learning. In the pioneering wall turbulence simulation of Lee et al. [15], a feedback law for skin friction reduction was learned in model-free manner employing a neural network. The current study continues these works and employs genetic programming as powerful regression technique for the optimization of the multiple-input multiple-output control laws.

In the following, we discuss in detail the different approaches on the example of turbulence. The paper is organized as follows: In section II, we discuss the methodological approach; we present our novel model-free approach based on GP for controlling complex and turbulent flows. We motivate the use of GP and describe the algorithm, called Genetic Programming Control (GPC). Then in section III we show the possibilities of GP-based model-free control by using it to stabilize a non-linear system exhibiting frequency cross-talk between two oscillators and to enhance the chaos in a periodic Lorenz system. In section IV we display the successes obtained on experimental demonstrators including the mixing enhancement of a turbulent shear-layer, recirculation reduction on a backward facing step using real-time PIV and separation control on a separating turbulent boundary layer. Finally, we discuss the advantages and drawbacks of the methods and conclude in section V.

II. METHOD

In this section, we review control approaches to turbulent flows with reference to applications in that field (section II A). Then, we describe the control algorithm we chose, where

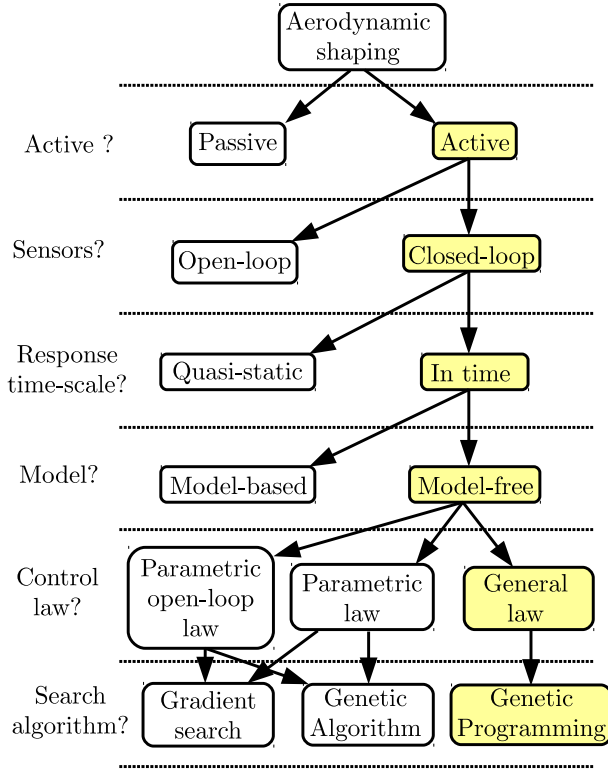


FIG. 1. Decision tree for control design.

genetic programming serves to identify the control law (sections II B–II H).

A. Approaches to control turbulent flows

1. Passive or active control

Many systems have to be modified to improve their aerodynamics performance for off-design situations. These modifications are to be achieved as to not disrupt the main design objectives of the system. To this end, the field of flow control developed many strategies to control flows [16–20]. The nature of the system as well as the objective of the control can guide the designer’s choices while going through a decision tree as depicted in Fig. 1. Passive control is on many aspects (design and maintenance cost, failure odds e.g.) the easiest way to achieve flow control. It consists in adding small passive devices or slightly change the geometry of the system (e.g. deflectors, vortex generators) in order to reach better flow

configurations [21]. The first drawback of passive control is that it impacts the system under all operating conditions while the control is only required in particular circumstances. For instance the design of wings is strongly impacted by take-off and landing conditions, which may lead to designs that are sub-optimal for cruising flight conditions. Active control, on the other hand, brings as a first advantage the ability to turn the control off or on. Then, active control can be characterized by its energy and frequency content. Injecting energy improves control authority and makes it possible to reach states which are unreachable with passive control. Finally, the possibility to add a frequency content allows to act on specific flow mechanisms of the flow (usually through instabilities) which leads to higher control efficiency.

2. *Open- or closed-loop control*

Open-loop control has been successful in many applications. Typical instances are periodic forcings based on flows' most sensitive frequencies. [8, 22]. Though, such a design can only be effective using information that is available during the design phase of the control. Should the flow conditions change, then the control can turn totally ineffective or can hurt the system performances. A way to solve this difficulty is to provide information from sensors to the controller leading to a closed-loop control design [2, 10, 23–25]. The sensors can be used to evaluate global quantities characteristic of the flow e.g. Reynolds number, temperature, static pressure, noise level or more specific values e.g. instantaneous velocities, dynamical pressure, perturbations. The feedback loop can be employed both to evaluate the flow state and the control efficiency which can in turn be used to refine the control design. Thus, closed-loop control can exhibit an intrinsic robustness when compared to open-loop control [2, 10]. Furthermore the access to dynamical information on the system allows to target the trajectories of the dynamical system.

3. *Adaptive or in-time closed loop control*

Adaptive control builds on slow adaptation of parameters of a working open-loop control, like amplitude or frequency of periodic forcing. It can be implemented by exploiting statistical information of the system to evaluate a deterioration in the control performance

and to correct the control parameters accordingly. It has been successfully implemented on many applications [26–28], exhibiting remarkable gains in robustness against changes in the flow conditions. Adaptive control can be schematized by a succession of effective open-loop controls which are selected according to the monitored flow conditions. While it has the advantage to allow a comparatively simple technical implementation thanks to modest needs in the information flow frequency (which is usually one to several order of magnitude lower than the frequency content of the relevant physics), its absolute performance can never be better than the selected open-loop controls for the considered flow conditions. By allowing an in-time information flow to the controller, the system trajectory can be inferred in real-time, opening the way to stabilization (e.g. opposition control [29], time-delay synchronization [30]). This class contains the pioneering simulations on skin friction reduction of wall turbulence — either by stabilizing the whole flow in time [31] or by preventing streaks in the laminar wall region [29, 32, 33].

4. *Model-based or model-free*

Monitoring trajectories and acting accordingly based on incomplete sensor information is a challenging task. A first approach is to use a model of the flow to both build an observer that can estimate the state of the flow based on the information of the sensors, and compute the best possible actuation that leads the modeled flow in the desired state. As most flows correspond to high-order dynamical systems inadequate for real-time control, a control-oriented reduced-order model (ROM) is useful as test-bed for the understanding of the actuation mechanism and as enabler for online-computability in an experiment. The tools developed in the linear control theory have demonstrated numerous successes on flows that are well adapted to model reduction [34–37]. With increasing flow complexity, this task becomes exponentially difficult if at all possible. These challenges are summarized in Fig. 2 and detailed below. The next step would be to assess if a model of the flow is needed or at all available. The sparse experimental literature on in-time turbulence control can be explained with the immense modeling and control challenges.

Many laminar shear flows generally exhibit one or few frequencies. The actuation frequency (input) is recorded by the sensors (output). Steady solutions at low Reynolds numbers have an actuation dynamics which may approximately satisfy a *superposition principle*:

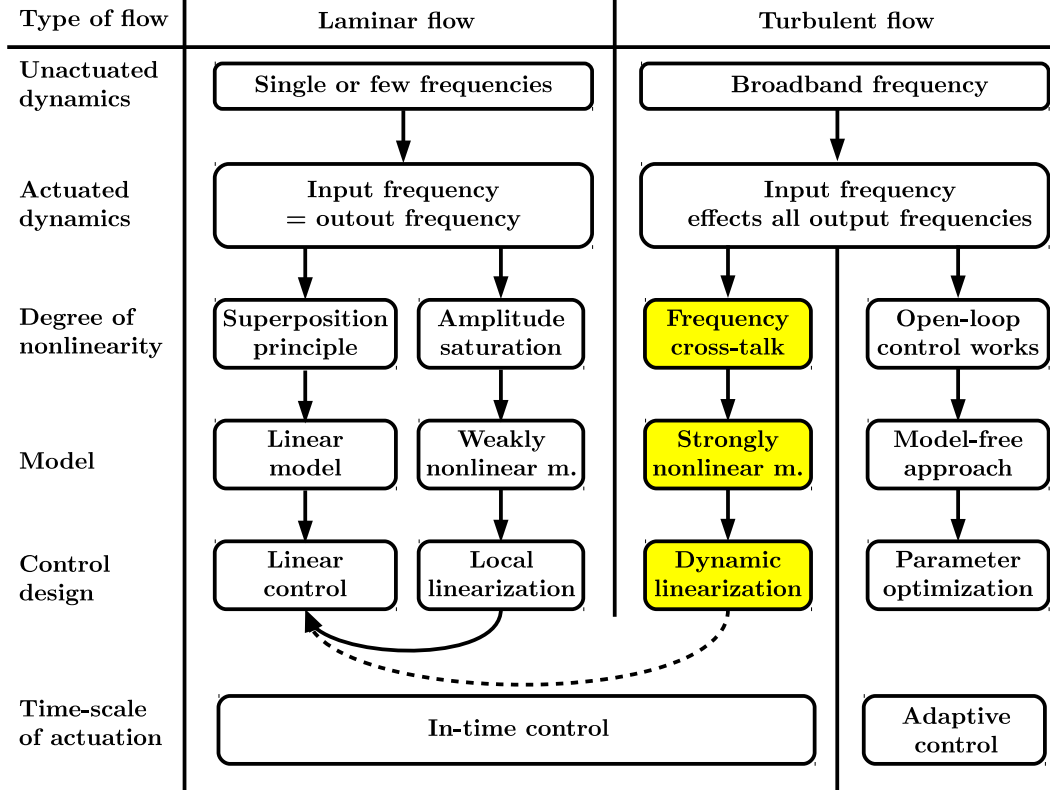


FIG. 2. Physics of closed-loop turbulence control strategies with considered key challenges marked in yellow. For details see text.

The sum of two actuations yields the sum of the individual responses in the sensors. In this case, a linear model approximates the flow behavior and can be used for linear control design. At larger Reynolds number, flow mechanisms like periodic vortex shedding have evolved due to an instability and it are limited by a nonlinear amplitude saturation mechanism. In this case, the superposition principle is no longer valid, but control may be based on a locally linearized model parameterized by the base flow [38, 39].

Continuing with the right side of Fig. 2, turbulent flows exhibit a broadband frequency dynamics seen by the sensors already in the natural case. A monochromatic forcing changes the whole spectrum due to frequency cross-talk, like vortex pairing (inverse cascade) or the normal cascade to smaller structures. It is a non-trivial challenge to frame this actuation response in a reduced-order model. In many studies, a periodic forcing is optimized in a model-free adaptive manner (see right-most column of Fig. 2). In some cases, the nonlinear frequency cross-talk can be modeled and exploited for a 'dynamic linearization' leading to

a linear control law. One example is a turbulent flow dominated by a natural shedding frequency and mitigated by a different actuation frequency [7]. In this case, the flow can be described by generalized mean-field model [9]. For more complex dynamics, the search for a model-based control strategy constitutes a large challenge. An attempt at obtaining a linear model for the mixing layer flow presented in section IV A is described in Appendix. The sparse experimental literature on in-time turbulence control can be explained with these immense modeling and control challenges.

On the other hand, a model-free approach can be investigated where the control law has to be determined based on realizations of the process.

5. *Choice of a control law*

Following Fig. 1, the next decision would be on whether the control law should be found by parameter identification of an open-loop forcing, e.g. periodic forcing, or by parameter tuning of an in-time control law, e.g. PID feedback, or by structure identification of a general law. Parametrization of open-loop forcing could correspond e.g. to the determination based on the flow condition of the optimal frequency, amplitude and duty cycle of an harmonic forcing. A parametric law could correspond to the determination of the coefficients of the Taylor expansion of a control law, or the weights of the connections in a neural network. Determining a general law is the ultimate grail as this determination is likely to reveal the enabling, possibly nonlinear mechanism which allows an effective control. This is the approach considered in this contribution.

The search algorithm used to obtain the control law is crucial. A gradient search method is able to find local minima around the initial operating conditions. Examples are extremum and slope seeking as well as auto-tuning PID controllers. The known drawback of gradient-based methods is their high sensitivity to local minima. Depending on the topology of the cost function graph, this can lead to the inability to find a sufficiently efficient control or to hysteresis under changing conditions. On the other hand, evolutionary algorithms like genetic algorithms (GA) and genetic programming (GP) have both exploration and exploitation mechanisms. GA and GP which belong to the machine learning field can be used for solving global optimization problems, giving the possibility to find a global extremum even when a local minimum is first detected. It is to be noted that Monte Carlo methods also

treat successfully the local minima problem but those methods lack an effective exploitation phase which leads to prohibitive costs for a large search space. Machine learning methods have been used successfully to derive micro-controllers for robotics, computer programs, optimal shapes, image recognition and data-based decision to name a few examples [40]. The evolutionary algorithms from machine learning have the advantage of being most performant when the search space is complex, especially when numerous local extrema are present. As GA can only optimize parameters, it cannot be used to derive the expression of a general function, but can be employed as a search algorithm in a neural network. GP is able to construct general functions from a set of given user-defined functions. This algorithm is the best candidate to determine in-time closed-loop control laws in a model-free framework.

B. Genetic Programming Algorithm

This algorithm relies on GP to derive feedback control laws for any complex dynamical system. In the sequel this algorithm is called Genetic Programming Control (GPC). We first formulate the control problem in a dynamical system's framework. After motivating the use of GP with respect to other popular evolutionary algorithms, we present the different steps used in the GP process to determine an effective control law. Finally we discuss the choice of parameters as well as the stopping criteria used to judge the convergence.

In the following, we restrict the description to ordinary differential equations for reasons of comprehensibility. The generic multiple-input/multiple-output (MIMO) system is represented in phase space by the vector $\mathbf{a} \in \mathbb{R}^{n_a}$, the state is monitored by sensors $\mathbf{s} \in \mathbb{R}^{n_s}$, and controlled by actuators $\mathbf{b} \in \mathbb{R}^{n_b}$,

$$\frac{d\mathbf{a}}{dt} = \mathbf{F}(\mathbf{a}, \mathbf{b}) , \quad \mathbf{s} = \mathbf{H}(\mathbf{a}) , \quad \mathbf{b} = \mathbf{K}(\mathbf{s}) , \quad (1)$$

with \mathbf{F} denoting a general nonlinear function, \mathbf{H} the measurement function, and \mathbf{K} the sensor-based control law. This law minimizes the state- and actuation-dependent cost function:

$$J = J(\mathbf{a}, \mathbf{b}). \quad (2)$$

The cost function value grades the performance of a control law $\mathbf{K}(\mathbf{s})$. The lower the value of the cost function, the better the control law solves the problem.

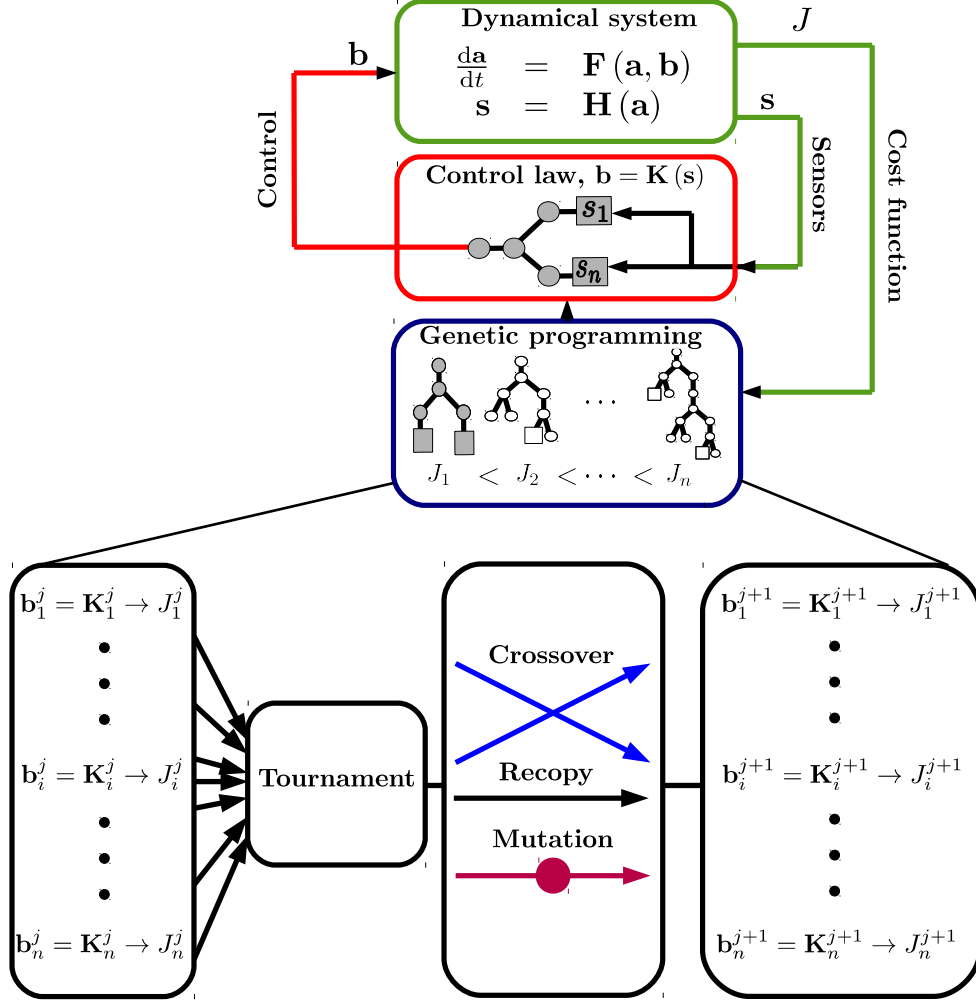


FIG. 3. Model-free control design using GP. During the learning phase, each control law candidate is evaluated by the dynamical system or experimental plant. This process is iterated over many generations of individuals. At convergence, the best individual with the smallest cost function value (in gray in the GP box) is used for control.

C. Genetic Programming

For determining the expressions of the optimal feedback laws, potential algorithms naturally arise from machine learning [41]. Machine learning algorithms can be roughly categorized as classification, regression, clustering and dimensionality reduction. Three algorithms are especially suited to learn a system behavior from data: Genetic Algorithms (GA), Artificial Neural Network (ANN) and Genetic Programming (GP). GA [42] and ANN [15] have

both been used in a control context, though, without structure identification of the control law. A GA can only guess parameter values, and as such, can only be used assuming the structure of the control, which is obtained by other means before optimization. An ANN is, strictly speaking, the arrangement of perceptrons, their interaction and their contained sigmoid functions that define an input/output system. The algorithm used to learn the weights of the links between perceptrons can be freely chosen among a pool of methods. If the algorithm is not evolutionary (e.g. reinforcement by error back-propagation in the hidden layers, the most common algorithm used), then the learning algorithm is similar to a gradient-based search, which means that the process is sensitive to local extrema of the search-space. If the algorithm is evolutionary (e.g. GA) then we obtain a process which is able to deal successfully with local minima. The main drawback with ANN is that it is hard to process the result analytically. GP, however, yields a function as a possibly nonlinear combination of mathematical operations. Consequently, one can read and analyze the result in order to understand how the learned mechanism actually works. The evolutionary principle is the same as in genetic algorithm, combining phases of search-space exploration and phases of convergence toward an extremum. Whereas GP has been extensively used to design computer programs [43], micro-controllers in robotics [44, 45] and to identify a dynamical system [46–48], to our knowledge, it has never been used before in a model-free control strategy for dynamical systems. In the following sections, we detail our approach (Fig. 3).

D. First generation

The flowchart for GP is given in Fig. 4: a set (called population) of n control laws (called individuals) is evolved by genetic operations: replication, crossover and mutation. Selection is achieved with respect to how the individuals behave in terms of minimization of the cost function J . In the following we detail the algorithm, beginning with the population creation. Several representations can be used to manipulate functions inside the Genetic Programming algorithm (e.g. trees, linear programming). We chose to use a tree-like representation. The two main advantages of this representation are to be human-understandable at a glance, and to be easily synthesized and manipulated with a computer. We note that, due to the recursive structure, functional programming languages are particularly well suited, as

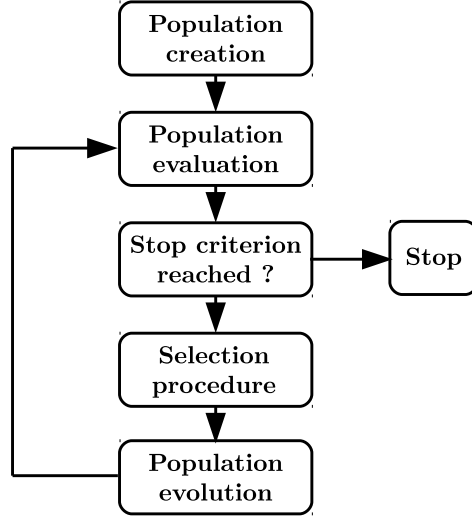


FIG. 4. Flowchart for the Genetic Programming algorithm.

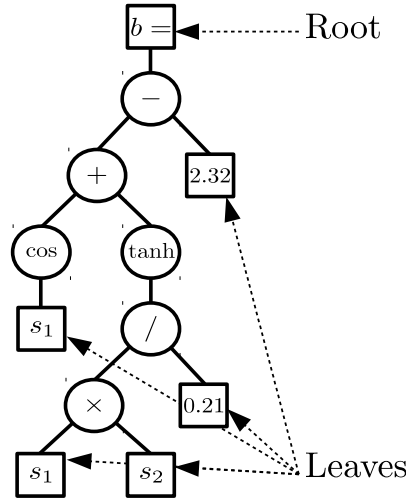


FIG. 5. An expression tree representing the function $b(s_1, s_2) = \cos(s_1) + \tanh((s_1 \times s_2)/0.21) - 2.32$.

Haskell or LISP.

In the GPC algorithm, the control laws exist as expression-trees (see Fig. 5). Expression-trees are usually built from a number of elementary functions which can take any number of arguments but return a single value. Typical examples are $(+, -, \times, /, \sin, \tanh, \dots)$. Here, the arguments of these constituents are sensors values, constants or outputs of sub-trees. A tree can be built from the final output value (the root of the tree) to terminal elements that can take no argument (the leaves). A typical visual representation of an expression-tree can

be seen in Fig. 5. This tree represents the function:

$$b(s_1, s_2) = \cos(s_1) + \tanh((s_1 \times s_2)/0.21) - 2.32, \quad (3)$$

where s_1 and s_2 represent the time-varying sensor values. A useful way to handle an expression tree is to write it recursively, e.g. in Polish notation; Equation (3) is then written as

$$(- (+ (\cos s_1) (\tanh (/ (\times s_1 s_2) 0.21))) 2.32).$$

Though less understandable by the human eye, recursive algorithms can generate, derive, manipulate and evaluate these expressions.

The generation process of any individual starts at the root. Then a first element is chosen from the pool of admissible elements composed of the basis functions and operators, the sensors and the constants. If a basis function or operator is chosen, new elements are added as arguments, the process is iterated to include their arguments until all branches have leaves. This process is subject to limitations. A given tree-depth (the maximum distance between a leaf and the root) can be prescribed by preventing the last branch to generate a leaf before aforementioned tree-depth and by enforcing the generation of leaves when the tree-depth is reached. Similarly it is possible to ensure that each branch reaches the same given tree-depth which generates a full density tree. GPC can implement any kind of these distributions, from a fully random generation to a given tree-depth distribution with given proportion of dense and less dense individuals. The first generation starts with a distribution of rather low tree-depth (2 to 6) and a 1 : 1 distribution of dense and less dense individuals. This choice generally ensures enough diversity for the creation of next generations. The initially low tree-depth takes into account that the genetic operations (see section II G) have a tendency to make the trees grow. This phenomenon is known as bloating of the trees. Though not used here, we note that it is also possible to manipulate the content of the trees by weighting the probability of each element. We do not introduce a priori control laws, though in principle possible. To enforce diversity, an individual is discarded if it already exists in the current population.

E. Evaluation

After the first generation of the population is generated, the individuals are evaluated according to the value of J which is computed for each individual. Usually, the evolution of the dynamical system is evaluated over a sufficiently long time interval and J is the value of an integral. If the evolution equation of the dynamical system is known, then this can be achieved through integration (e.g. Navier-Stokes solver). If the system is experimental then the value of J is computed by means of recording relevant data throughout one instance of the experiment using the individual under consideration.

As imposed by the selection process presented in section II F, the only restriction to the design of the cost function is that a ranking of the values of J assigned to each individual must be possible. This allows the use of expressions which might not be differentiable or even continuous. As stated in section II B, the algorithm assumes that J is positive and must be minimized. Also for the sake of comparison with other control methods like Linear Quadratic Regulator, it is convenient to design the cost function in a similar way. In the case of several objectives, one can use regularization by adding more objectives using a parameter, the control law contains appropriate weights:

$$J = J_a + \gamma J_b, \tag{4}$$

where J_a is a given measure on the system that need to be minimized in order to achieve the control objectives and J_b a value associated with the cost of actuation. More objectives can be added, e.g. complexity of the function. The coefficient γ is determined in order to give priority to some objectives. A rule of thumb is to set it in order to establish admissible trade-offs between the objectives.

One big problem in the correct evaluation of the cost function consists in the integration time needed to gather sufficient statistics for an accurate evaluation of J . This applies in the first place to experimental runs, which might be expensive. If the recording time is not long enough, the evaluation of one individual does not necessarily return the same cost function value for all instances. Of course, that holds for huge numerical runs, too.

So, a large error on the value, either due to a measurement error or an exceptional performance of a non robust control law can lead to mistakenly grade a control law. As explained below, the search-space is primarily explored around the individuals that perform

the best. If an intrinsically low-performing individual gets a mistakenly good evaluation, then the whole process is endangered. This is why a reevaluation process is implemented on the best individuals. Here, the five best individuals are evaluated five times, and their cost function is averaged. This procedure ensures that the best performing individuals are more carefully evaluated to avoid that the search process gets stirred in a wrong direction.

F. Selection

After the evaluation, the population evolution starts: each individual is subject to one of the four “genetic” operations, reproduction, crossover, mutation, elitism, explained in detail below in section II G. This process is called tournament. Each time an individual needs to be selected for a genetic operation, a number n_p of individuals is randomly chosen from the last evaluated generation. From this set of individuals, the one with the smallest cost function value is selected. As the population size n is fixed, n selection tournaments are run each time a new generation is created. The population is ranked by decreasing cost function value. Because typically $n \gg n_p > 1$, the probability for individual i to win a tournament is $((n-i)/(n-1))^{n_p-1}$. On average, each individual enters n_p tournaments, so if an individual has a rank of $x \times n$ in the generation ($x \in [0 \dots 1]$), then its “genetic content” contributes roughly to $n_p \times (1-x)^{n_p-1}$ new individuals. A typical value for $n_p = 7$ only the first half of the ranked generation contributes to the next generation. Individuals which are ranked higher can still contribute, but these are rare events. In this selection procedure, choosing n_p sets the harshness of the selection. On the other hand, this selection procedure does not take into account the global distribution of the cost function values like in other selection processes such as a fitness proportional selection. If an individual performs much better than the rest of the population it does not have better selection chances than an individual that barely performs better than the rest of the population. This choice ensures that the diversity in the population does not disappear too fast with an associated trade-off in convergence speed.

G. Genetic operations

Typically, genetic operations comprise *elitism*, *reproduction*, *mutation* and *crossover*.

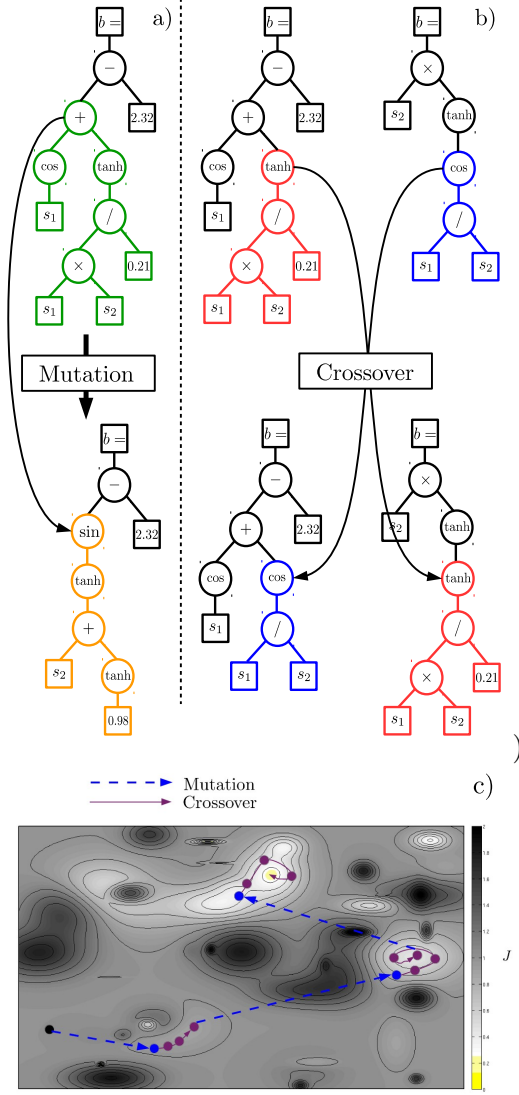


FIG. 6. Top: example of mutation (a) and crossover (b) genetic operations. Bottom: Schematic representation of the search-space exploration by mutation (dashed lines) and crossover (solid line) by following one branch of the genealogy of the best individual (c). While mutations enable large-scale exploration, crossovers support the convergence to local minima.

Elitism consists in copying the n_e best individuals of the evaluated population to the next generation. This operation ensures that the best control laws stay in the population. Once the elitism process is finished, $n - n_e$ individuals are generated through reproduction, mutation and crossover. The probability of these processes is respectively P_r , P_m and P_c with $P_r + P_m + P_c = 1$. *Reproduction* copies the selected individual to the next generation. This is analogue to elitism, with lower rank, though. *Mutation* replaces an arbitrarily chosen subtree by a randomly generated new subtree. For that, the same procedure is used as in

the creation of the first generation. Finally, *crossover* uses two selected individuals and exchanges one randomly chosen subtree between them.

Reproduction ensures a certain stability of the convergence process: it guarantees that a part of the population stays in the vicinity of explored local minima of the search-space, keeping potentially useful individuals in the population, exploiting them further before they are discarded. Crossover and mutation are responsible respectively for the exploitation and exploration of the search-space. As we progress among the generations, the probability to cross similar individuals increases: the best individual propagates its genetic content n_p times on average. If this content allows the concerned individuals to stay in the first positions of the ranking, it is replicated about $n_p^k \times P_c$ times after k generations. Then crossovers of individuals selected in the top of the ranking soon cross similar individuals and explore the vicinity of the dominant genetic content. On the other hand, mutations introduce new genetic content in the population, hence allowing large-scale exploration of the search-space. Mutation and crossover operations are displayed in Fig. 6. Figure 6 (bottom) illustrates how an evolutionary algorithm explores the search-space for a bi-dimensional problem with local minima: the association of these operations allows to explore around the local minima found while still exploring the search-space for better solutions.

H. GP parameters and stopping criteria

There are no general rules to choose optimal parameters of evolutionary algorithms. A common practice is to check the optimality of the solution offered by GP by reproducing the process a number of times using different sets of parameters. This way, a guiding statistics can be obtained. The main impact of the parameters' modifications is on the ratio between exploitation (i.e. convergence) and exploration of the search-space. Monitoring the evolution of the evaluated populations is the best way to fine-tune the GPC process. Now, we discuss the role of the GPC's parameters:

- Population size n : the more individuals define the first generation, the more the search-space is explored from the initialization. On the other hand, a large initial population implies more evaluation time without any convergence. Let us consider 1000 evaluations. If only the first generation is evaluated, then it is equivalent to a Monte Carlo process. Alternatively, one could devote 1000 evaluations to 20 generations with 50

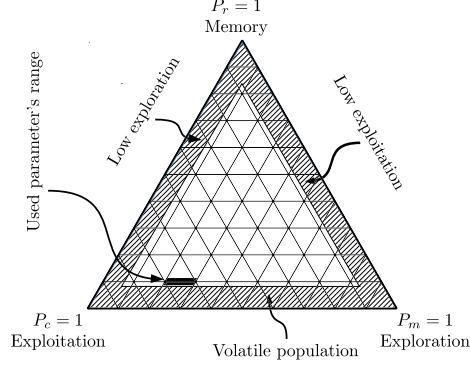


FIG. 7. Probability selection for the genetic operations. Each line parallel to the opposite side of a summit removes 10% of probability to the genetic operation considered at the summit. There is no globally optimal parameter selection. Depending on the problem, diversity or convergence needs to be modified. The range of parameters used in the present study is represented by the black area. Hatched areas represent the parametric space where essential aspects of the evolutionary algorithm are endangered with volatile population translating in previous winning options to be forgot once a better solution is found (not enough replications), low convergence (not enough crossovers) or low exploration (not enough mutations).

individuals. This implies 20 convergence steps around the best found individuals.

- Genetic operation probabilities (n_e , P_r , P_m , P_c): Elitism is encouraged as it ensures that the n_e best performing individuals remain in the population. Reproduction, mutation and crossover probabilities parametrize the relative importance between exploration and exploitation (Fig. 7). A large mutation rate P_m implies large-scale exploration and thus population diversity. If all individuals in the population share a similar expression and have almost similar costs then the mutation rate has to be increased. On the other hand, the crossover rate influences the convergence speed. If the individuals stay all different and the cost function value histogram does not show a peak around the lowest value, then the convergence is not sufficiently rapid and the crossover probability needs to be increased. Finally the reproduction ensures that a given search-space area is explored during a certain number of generations. This parameter ensures as well diversity and exploration of different areas.
- The selection procedure influences also the diversity of the following generation. The

number of individuals n_p that enter a tournament directly influences the number of individuals that contribute to the next generation. Reducing n_p increases the diversity while increasing it accelerates the convergence.

- The choice of the elementary functions is intimately linked to the problem at stake. This question should be determined in concordance with the choice of the sensors and actuators.
- The maximum number of generations to compute is eventually determined by the available testing time. A stopping criterion can end the iterations prematurely, for instance if the optimal solution is reached ($J = 0$) or if the average and minimum of the cost function distribution do not evolve anymore.

Often, a high rate of crossover and a low rate of mutation is favored to allow for a large variety of a initial population as large as possible. However, as stated above, the concrete choice is problem-dependent. With such choice a fast convergence may be obtained, and are particularly suited if the evaluations can be parallelized. In experiments, however, the total time of evaluation is critical and one cannot afford a large population. Furthermore the uncertainties in the cost function evaluation do not allow to achieve a convergence below the measurement error. The best experimental compromise found was to deal with reduced populations (in the order of 50 to 500 individuals) associated with a high mutation rate (from 25 to 50%). Though we decided to keep these values constant during the course of each experiment, further performance improvement can be achieved by adapting them with respect to the phase (exploration or exploitation) of the learning process.

III. LOW-DIMENSIONAL NON-LINEAR DYNAMICAL SYSTEMS

To demonstrate the usefulness of GPC for solving non-linear problems, GPC is first used on low-dimensional state-space systems exhibiting key non-linear features. The first one is an oscillator model displaying frequency cross-talk (section III A), the second one is a Lorenz system characterized by chaotic trajectories (section III B). GPC is used to find control laws for both systems, either to achieve stabilization or to optimize the Lyapunov exponents on the Lorenz system. These numerical systems have also served as playground for testing the

influence of the different GPC parameters on convergence (section III C) and infer usable parameters for the experiments (section III D).

A. Oscillator model

In order to demonstrate nontrivial results in our control, we consider a simple system with frequency cross-talk between a natural unstable oscillator and an actuated stable oscillator via the base flow. This model can be derived from the Navier-Stokes equation [9] and has successfully described the actuation effect of high and low-frequency forcing of flows around airfoils, bluff-bodies and of swirling jets. It is arguably the most simple dynamical system prototype exhibiting frequency cross-talk. This model can be viewed as a generalization of the Landau model for the bifurcation from equilibrium to a periodic oscillation. A similar model can be built as a reduced-order model for a cylinder wake flow. As GPC is intended to be used on strongly non-linear systems where frequency cross-talk severely limits the use of a linear framework, we want to demonstrate the technique on the following model:

$$\frac{d}{dt} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \sigma_1 & \omega_1 & 0 & 0 \\ -\omega_1 & \sigma_1 & 0 & 0 \\ 0 & 0 & \sigma_2 & \omega_2 \\ 0 & 0 & -\omega_2 & \sigma_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix} \quad (5)$$

with $\sigma_1 = \sigma_{10} - (a_1^2 + a_2^2 + a_3^2 + a_4^2)$.

Hereafter, we denote the sum of squared amplitudes as energy to avoid linguistic sophistication. We set $\omega_1 = \omega_2/10 = 1$ and $\sigma_{10} = -\sigma_2 = 0.1$ so that the first oscillator (a_1, a_2) is unstable at the origin while the other one (a_3, a_4) is stable. When uncontrolled ($b \equiv 0$), the nonlinearity drives the first oscillator to nonlinear saturation by the change of total energy. The actuation directly effects only the stable oscillator. We choose to stabilize the first oscillator around its fixed point $(0, 0)$ and thus a cost function which measures the fluctuation energy of that unstable oscillator. For any useful application, the energy used for control is required to be small, hence, we penalize the actuation energy. The cost function is then defined as:

$$J = \langle a_1^2(t) + a_2^2(t) + \gamma b^2(t) \rangle_T, \quad (6)$$

with $\gamma = 0.01$ as penalization coefficient and $\langle \cdot \rangle_T$ denoting the time-average over the interval $[0, T]$. Here, $T = 100 \times 2\pi/\omega_1$ is chosen to allow meaningful statistics. The quadratic form

TABLE I. GPC parameters used for the control of model (6).

Parameter	Value
n	1000
P_r	0.1
P_m	0.3
P_c	0.6
n_p	7
n_e	1
node functions $+$, $-$, \times , $/$, \sin , \exp , \log , \tanh	

of the state and the actuation in the cost function is a standard choice in control theory. We apply GPC with full-state observation ($\mathbf{s} \equiv \mathbf{a}$) to exploit all potential nonlinear actuation mechanisms stabilizing the first oscillator.

Knowing the nonlinearity at stake, an open-loop strategy can be designed: exciting the stable oscillator at frequency ω_2 provokes an energy growth which stabilizes the first oscillator as soon as $a_3^2 + a_4^2 > \sigma_{10}$. Note that the linearization of (5) yields two uncoupled oscillators. Thus, the first oscillator is uncontrollable in a linear framework.

1. Results

The function space is explored by using a set of elementary ($+$, $-$, \times , $/$) and transcendental (\exp , \sin , \ln and \tanh) functions. The functions are 'protected' to allow them to take arbitrary arguments in \mathbb{R} (e.g. a thresholding is achieved on denominators in divisions to avoid division by zero). Additionally, the actuation command is limited to the range $[-1, 1]$ to emulate an experimental actuator. Up to 50 generations comprising 1000 individuals are processed. The tournament size is $n_p = 7$, elitism is set to $n_e = 1$, the probabilities of replication, crossover and mutation are $P_r = 0.1$, $P_c = 0.6$ and $P_m = 0.3$ respectively (see Tab. I).

The control law ultimately returned by the GPC process corresponds to the best individual of the last generation. The performance and the behavior of the control law are displayed in Fig. 8. This control law is energizing the second oscillator up to $10^0 \gg \sigma_{10}$ as soon as

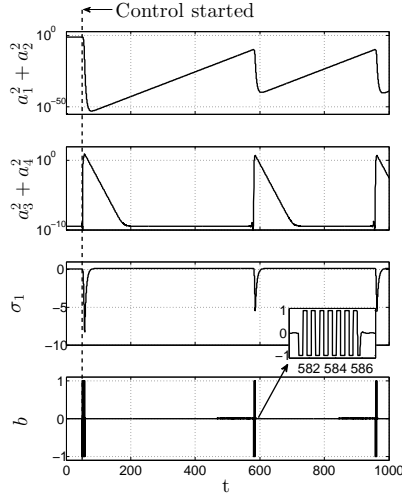


FIG. 8. Genetic programming control of the model. When the energy contained in the first oscillator (top) is larger than 10^{-10} the control (bottom) is exciting the second oscillator at frequency ω_2 , its energy grows so that σ_1 reaches approximately -5 . This results in a fast decay of the energy in the first oscillator after which the control goes in “standby mode“. An animation of the controlled system can be found in supplemental video S1.

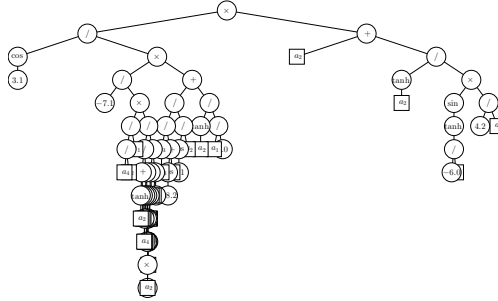


FIG. 9. Tree representation of the best individual obtained by GPC for the control of model (6).

the first oscillator has an energy which is larger than 10^{-10} . This is stabilizing the unstable oscillator very quickly with an amplitude decay scaling as $10^{-0.1t}$. After stabilization, the control amplitude stays at very low values. That keeps the stable oscillator at a correspondingly low energy ($\approx 10^{-10}$), while the amplitude of the unstable oscillator is exponentially increasing with its initial growth rate σ_{10} . This control law exploits the frequency cross-talk and vanishes when not needed, i.e. $a_1 \approx a_2 \approx 0$. The expression tree of the best solution

found by GPC can be viewed in Fig. 9. It can be summarized as follows:

$$\begin{aligned}
b &= K_1(a_4) \times K_2(a_1, a_2, a_3, a_4) \quad \text{with} \\
K_1(a_4) &= 5.475 \times a_4 \quad \text{and} \\
K_2(a_1, a_2, a_3, a_4) &= \frac{\left(\frac{(\frac{a_4}{4.245}) \times (\sin(\tanh(\frac{a_4}{-5.987})))}{\tanh(a_2)} + a_2 \right)}{\cos(3.053)} \times \\
&\quad \left(\frac{8.965}{a_1} + \frac{\frac{a_2}{\cos(3.053)}}{a_4 + \frac{\cos(-8.208)}{\log(a_3)}} \right) \times \\
&\quad \left(\frac{\left(\frac{\frac{a_1}{a_4}}{(\frac{\sin(a_4)}{-6.912 - (a_4)}) \times (\frac{a_1}{(a_2) \times (a_4)})} \right)}{\frac{a_2}{a_1} + \tanh(a_2)} \right) \times \left(\frac{a_1}{a_4} \right) \\
&\quad \left(\frac{\frac{a_2}{a_1} + \tanh(a_2)}{-7.092} \right). \quad (7)
\end{aligned}$$

The function $K_1(a_4)$ describes a phasor control that destabilizes the stable oscillator. The function $K_2(a_1, a_2, a_3, a_4)$ acts as a gain dominated by the energy of the unstable oscillator. That control could not be derived from a linearized model of the system. Moreover, less energy is used as compared to the best periodic excitation.

B. Lorenz system

One well-known feature of non-linear systems with dimension larger than three is their ability to display chaotic behavior. The control of such systems is a field in its own. Chaos being certainly unfavorable to many applications, many studies have investigated the possibility to stabilize chaotic trajectories in periodic orbits [30, 49, 50]. On the other hand, chaos can be sought in order to increase mixing [51]. We use GPC on a Lorenz system controlled in the third component in order to optimize the maximal Lyapunov exponent while keeping the system in a finite state space.

1. Problem formulation

The system is formulated as follows:

$$\frac{da_1}{dt} = \sigma(a_2 - a_1), \quad \frac{da_2}{dt} = a_1(\rho - a_3) - a_2, \quad \frac{da_3}{dt} = a_1a_2 - \beta a_3 + b \quad (8)$$

with full-state feedback $b = K(a_1, a_2, a_3)$, i.e. $\mathbf{s} \equiv \mathbf{a}$. With $\sigma = 10$ and $\beta = 8/3$, the Lorenz system can be stable, periodic or chaotic depending on the value of ρ . We employ $\rho = 20$,

TABLE II. GPC parameters used for the control of the Lorenz system

Parameter	Value
n	1000
P_r	0.1
P_m	0.3
P_c	0.6
n_p	7
n_e	1
node functions $+$, $-$, \times , $/$, \sin , \exp , \log , \tanh	

such that the uncontrolled system ($b \equiv 0$) is periodic. Like [52], we aim at maximizing the largest Lyapunov exponent λ_1 while penalizing the actuation power with a factor γ . If λ_1 is positive, the system is chaotic and well-mixing. We define the cost function, which should be minimized, as:

$$\begin{aligned}
J &= \exp(-\lambda_1) + \gamma \langle b^2(t) \rangle_T & \text{if } \sum_{i=1}^3 \lambda_i < 0, \\
J &\rightarrow \infty & \text{if } \sum_{i=1}^3 \lambda_i \geq 0,
\end{aligned} \tag{9}$$

where $T = 100$ is the integration time and $\lambda_1 \geq \lambda_2 \geq \lambda_3$ are the finite-time Lyapunov exponents (FTLE). These exponents are obtained by calculating the local Jacobian for each time step and integrating its eigenvalues along the trajectory. J is assigned the largest computable real number on the computer if the sum of the Lyapunov exponents is positive or the states exceed the bounds we specify.

2. Results

GPC is applied to the periodic Lorenz system to maximize the largest finite-time Lyapunov exponent while keeping the solution bounded. The basic operations that compose the control law are $(+, -, \times, /)$ as well as randomly generated constants. The maximum number of generations is chosen as 50 with 1000 individuals each. The tournament size is $n_p = 7$, elitism is set to $n_e = 1$, the probabilities of replication, crossover and mutation are respectively $P_r = 0.1$, $P_c = 0.6$ and $P_m = 0.3$ (see Tab. II). We consider for γ the values of $\gamma_S = 1$, $\gamma_W = 0.01$ and $\gamma_N = 0$, representing strong, weak and no penalization of the actu-

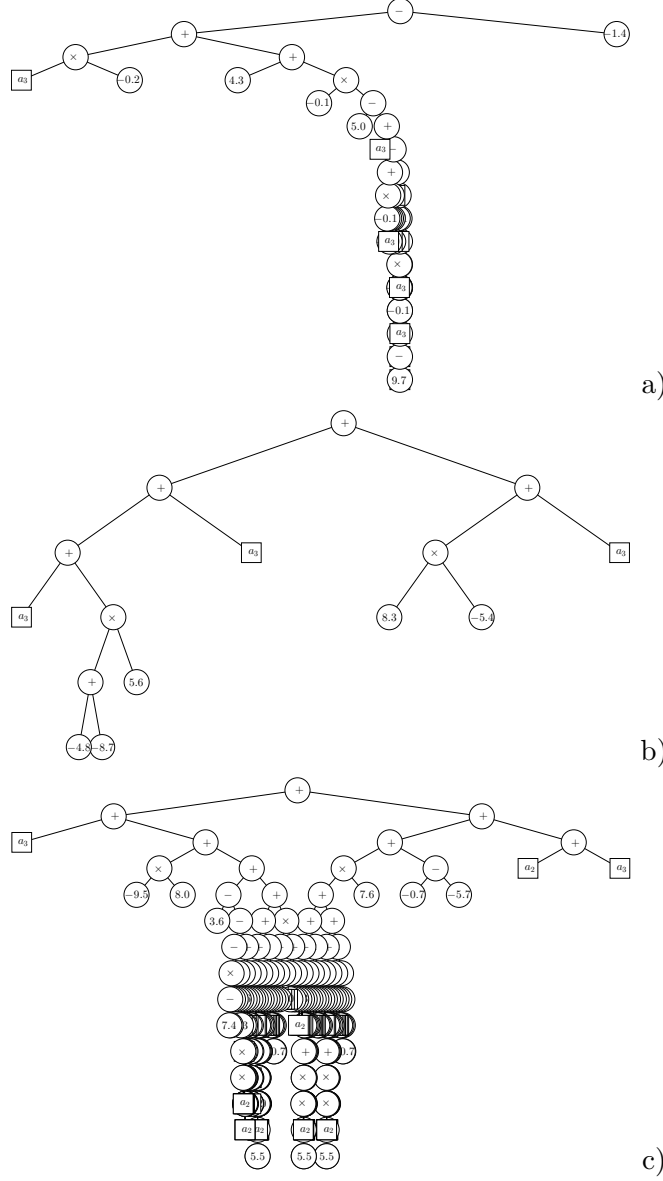


FIG. 10. Tree representation of the best individuals obtained by GPC for the control of the Lorenz system with $\gamma = 1$ (a), $\gamma = 0.01$ (b) and $\gamma = 0$ (c).

ation. This illustrates how the cost function definition influences the problem to be solved. After 50 generations, the best individuals (see Fig. 10) associated with strong, weak and no penalization have maximum FTLE of $\lambda_1 = 0.715$, 2.072 and 17.613, respectively. The changes in the system and the control function are displayed in Fig. 11. The control laws associated with γ_S and γ_W are affine expressions of a_3 and the reduction of the actuation cost leads to a larger amplitude of the feedback. In those cases, the most efficient control leads the system into behaviors close to the canonical Lorenz system ($\rho = 28$, $\lambda_1 = 0.905$).

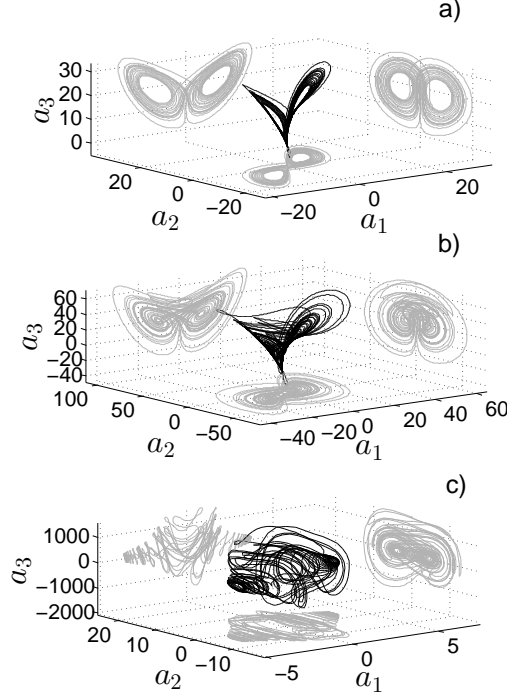


FIG. 11. Controlled Lorenz systems with $\sigma = 10$, $\beta = 8/3$ and $\rho = 20$. For $\gamma = 1$ (a), the system exhibits chaotic behavior ($\lambda_1 = 0.715$) close to the canonical chaotic Lorenz attractor with $\rho = 28$ ($\lambda_1 = 0.905$). For $\gamma = 0.01$ (b), the system exhibits more complex trajectories, the nature of the central fixed point has changed and $\lambda_1 = 2.072$. For $\gamma = 0$ (c), the nature of all fixed points has changed. The non-penalization of the actuation leads to a change in the scales ($\lambda_1 = 17.613$). An animation of the controlled system can be found in supplemental video S2.

For γ_W the nature (from saddle point to spiral saddle point) and the position of the central fixed point from the actuated system are changed. If the actuation is not penalized ($\gamma = 0$) the feedback law is a complex nonlinear function of all states. The nature and position of all fixed points are changed as λ_1 reaches higher values. or model-free approach has been proposed in the literature to optimize the largest Lyapunov exponent.

C. Convergence

Both examples illustrate how GPC is progressing towards the minimum of the cost function. The statistical process that selects the individual for breeding allows individuals which are not optimal to be selected. This keeps diversity in the population and ensures that the

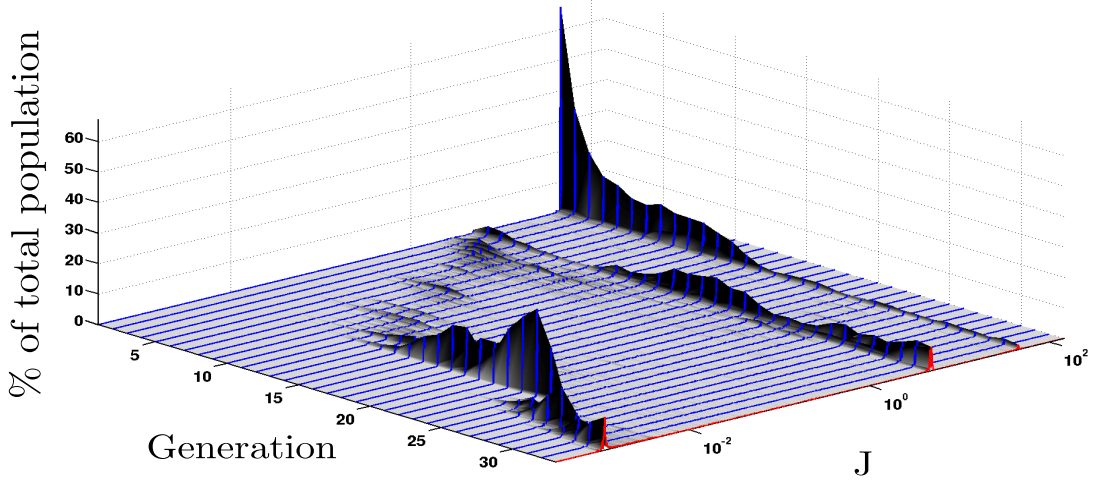


FIG. 12. Successive histograms of population repartition with respect to the cost function values for the stabilization of the oscillator model. High peaks indicate the high density of individuals around the cost function value. The histograms use 1000 bins which are logarithmically distributed between the first non-nul value and the maximum value of the cost function evaluations.

GPC process is not confined in a local minimum. In figure 12 successive histograms of the repartition of cost function values in the population are plotted for the model (6). The data comes from section III A but the aspect of the graph is typical of any evolutionary algorithm search. The population is stirred towards successive local minima. Around these minima an increasing number of individuals is exploring the search-space leading in a local repartition of cost function values due to the effect of crossovers. Mutations are mostly responsible of the jumps between local minima. Once a better local minimum is found, a large part of the population is shifted in that direction while other local minima are iteratively depleted. While a large part of the population lives around the minima, smaller ripples indicate the still existing diversity in the population. The two large *families* of individuals existing throughout the whole process account for obvious behaviors: individuals with null output (generated by a multiplication by 0 obtained by the difference of two identical subtrees e.g.) or with constant and saturated output. For the stabilization of our oscillator model, the GPC stopped after 35 generations, both oscillator energy and control energy vanished below numerical accuracy of the integration scheme (not represented in Fig. 12 due to the logarithmic scale).

D. Conclusions on usability of GPC and best practices for experiments

The stabilization of the simple oscillator model has shown that GPC is able to exploit strongly non-linear mechanisms to solve a control problem. The solution found is explicitly exciting one oscillator at his eigenfrequency to stabilize another oscillator at another frequency. This could not be achieved in a linear framework without the knowledge of the existing non-linearity. The chaotization of the Lorenz system highlights the fact that GPC can minimize a complex cost functional associated with a non-linear system. These GPC realizations on low-order systems allow to guide the parameters choices before applying GPC to experiments. The two main differences between these numerical test-cases and the experiments that will be presented in the next section are (i) the fact that the experiments are not parallelized and (ii) the fact that the cost function values are obtained with a measurement uncertainty. The total time of experiment being limited by the operation cost and facilities availability, limiting the number of individuals by generation is necessary. As for the second point, the uncertainty can be reduced (but not nullified) by increasing the evaluation time, which is not desired as each additional second of evaluation can easily translate in additional hours of total experimental time. This is partially solved by the re-evaluation process introduced in section II E. Nonetheless, an intrinsic noise on the experiment can never be avoided. This means that the intrinsic uncertainty level on the evaluation of the cost function is the utmost limit that can be reached while looking for minima of said cost function. This devalues the higher need of crossovers defended by GP experts, while a higher rate of mutation can alleviate the need of exploration and diversity induced by a reduced population. Hence, the parameters that will be used in experiments will be shifted towards high mutation rate and reduced populations.

IV. EXPERIMENTAL DEMONSTRATORS

GPC has been applied to four experimental demonstrators: in the TUCOROM mixing layer (section IV A), in the PMMH backward facing step flow (section IV B), in the LML separated boundary layer (section IV C) and in the PRISME separated boundary layer (section IV D). These four experiments are presented in Fig. 13.

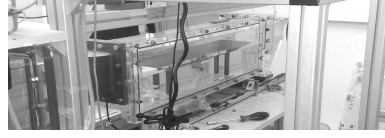
TUCOROM mixing layer



Facility: Wind-tunnel
Objective: Mixing enhancement
Sensors: Rake of 24 hot wires
Actuators: 96 pulsed jets
Reynolds number: 2000

a)

PMMH backward facing step



Facility: Water-tunnel
Objective: Recirculation reduction
Sensors: Real-time PIV
Actuators: Blowing or sucking jet
Reynolds number: 1350

b)

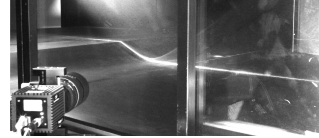
LML separating boundary layer



Facility: Wind-tunnel
Objective: BL reattachment
Sensors: 3 hot films
Actuators: Active Vortex Generators
Reynolds number: 13000

c)

PRISME separating boundary layer



Facility: Wind-tunnel
Objective: BL reattachment
Sensors: 2 hot films
Actuators: Active Vortex Generators
Reynolds number: 130000

d)

FIG. 13. Picture and global flow control parameters of the four experiments. a: TUCOROM mixing layer. b: PMMH backward facing step. c: LML separating boundary layer. d: PRISME separating boundary layer.

A. Control of a turbulent mixing layer

The first experimental implementation of GPC has been achieved in the TUCOROM mixing layer demonstrator (Fig. 13a). The goal of the experiment was to enhance the mixing properties of the mixing layer.

1. Experimental setup

The TUCOROM mixing layer demonstrator [53] consists in a dual turbine wind tunnel which allows to set two different velocities on each part of a splitter plate with a velocity ratio $U_1/U_2 = 3.6$ and Reynolds number based on the initial mixing layer thickness between 500

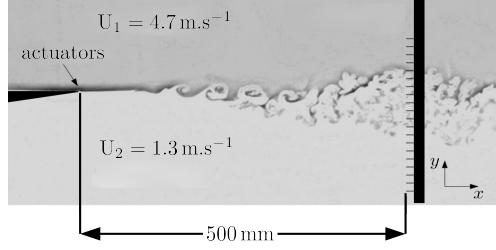


FIG. 14. Experimental configuration of the TUCOROM mixing layer. The hot-wire rake is placed 500 mm downstream of the separating plate to capture the structures in the shear layer. The vertical spacing of the hot-wire probe is $\Delta y = 8 \text{ mm}$.

(laminar) and 2000 (turbulent). The test-section is 3 m long with a subsection of $1 \times 1 \text{ m}^2$. Inside the splitter plate 96 micro-jets allow to blow in the streamwise direction. A rake of 24 hot wires, simultaneously acquired at 20 kHz, is placed at 500 mm downstream of the splitter plate. The sensors used for the genetic programming are the velocity fluctuations for 9 chosen, equi-distributed, sensors across the shear layer velocity gradient, while all 24 sensors are used for the evaluation of a given control law. GPC is applied on the configuration described in Fig. 14b with the following cost function J :

$$J = \frac{1}{W}, \text{ with } W = \frac{\langle [\sum_{i=1}^{24} s_i'^2(t)] \rangle_T}{\max_{i \in [1, 24]} (\langle s_i'^2 \rangle_T)}, \quad (10)$$

where $\langle \cdot \rangle_T$ is the average over the evaluation time $T = 10 \text{ s}$, s_i represents the hot wire signal from hot wire i , and s_i' its fluctuation calculated over $T/10$. W can be interpreted as the width of the fluctuation profile at the considered position. GPC is applied with $(+, -, \times, /, \sin, \cos, \exp, \tanh, \log)$ as node functions for the expression-trees. The parameters used are summarized in Tab. III.

2. Results

Many parameters for both the experiment and GPC have been tested [54]. We display the results for the configuration of Fig. 14 at the Reynolds number of 500 with the cost functional (10). Figure 15 shows the pseudo visualizations obtained for the natural flow, the best open-loop control (an harmonic forcing at frequency $f_a \approx 9 \text{ Hz}$ and duty cycle 50 %) and the GPC obtained control. Table IV displays the corresponding cost function values

TABLE III. GPC parameters used for the control of the TUCOROM mixing layer

Parameter	Value
n	1000 (first generation)
	100 (other generations)
P_r	0.1
P_m	0.25
P_c	0.65
n_p	7
n_e	1
Node functions $+$, $-$, \times , $/$, \sin , \cos , \exp , \log , \tanh	

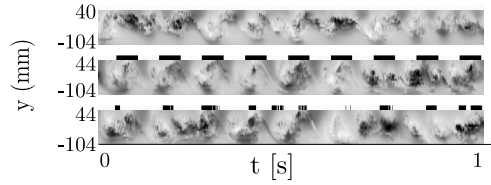


FIG. 15. Pseudo-visualizations of the TUCOROM experimental mixing layer demonstrator [53] for three cases: natural baseline (top, width $W = 100$ %), the best open-loop benchmark (middle, width $W = 154$ %), and GPC closed-loop control (bottom, width $W = 170$ %). The velocity fluctuations recorded by 24 hot-wires probes (see Fig. 14) are shown as contour-plot over the time t (abscissa) and the sensor position y (ordinate). The black stripes above the controlled cases indicate when the actuator is active (taking into account the convective time). The average actuation frequency achieved by the GPC control is comparable to the open-loop benchmark.

obtained for these three cases.

GPC yields a 70 % increase of the mixing layer width, which outperforms open-loop forcing by 16 %. Furthermore the actuation cost is 48% of what is used for the open-loop case (Tab. IV). This experiment also demonstrates that control laws obtained at different operating conditions are still performing, though not optimally, while open-loop controls can only be applied to given operating conditions. This robustness is directly due to the native retro-action implemented in closed-loop control. Further improvement could be achieved by

TABLE IV. Performance of uncontrolled, best open-loop controlled and GPC controlled systems. GPC manages to increase the width of the shear layer by 70%, outperforming the best open-loop control by 16%. Additionally, the actuation cost is 48% of the best open-loop control.

case	natural	best open-loop	GPC
J	1	0.65	0.59
W	100%	154%	170%
Actuation			
Cost	0	1	0.48

specifying the desired robustness inside the cost functional.

B. Control of a separated flow using Real-Time PIV

GPC has been applied in the PMMH water tunnel (Fig. 13b) on a separated flow over a backward facing step with the goal to reduce the recirculation zone.

1. Experimental setup

The PMMH water tunnel is gravity driven, allowing to reach speeds up to 22 cm.s^{-1} . In the current experiment (Fig. 16b) a backward facing step of height $h = 1.5 \text{ cm}$ is placed in the $L \times l \times H = 80 \times 15 \times 20 \text{ cm}^3$ test-section. The Reynolds number, based on the freestream velocity $U_\infty = 7.3 \text{ cm.s}^{-1}$, is $Re_h = U_\infty h / \nu = 1350$. Actuation is achieved thanks to a slotted jet located at a distance $2h$ upstream the step edge and can perform either blowing or suction. The angle between the jet and the wall is 45° in the direction of the flow. A real-time PIV system [55, 56] is used to compute flow fields at 42 Hz. The sensor signal used for GPC is the ratio between controlled and natural recirculation area of the flow obtained through instantaneous PIV fields, i.e.:

$$s(t) = \frac{A(t)}{A_u}, \quad (11)$$

where

$$A(t) = \int H(-u(t))(x, y) \, dx dy, \quad (12)$$

$$A_u = \langle A(t) \rangle, \text{ without actuation.} \quad (13)$$

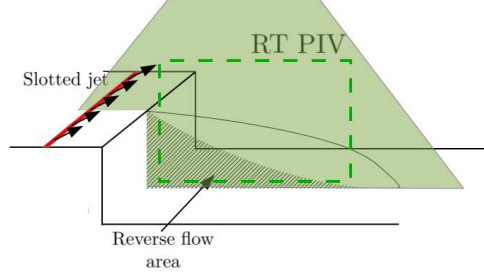


FIG. 16. Experimental configuration of the flow over a backward facing step in the PMMH water tunnel. A slotted jet is situated just upstream of the separation allowing to achieve blowing or suction in the boundary layer. Real-Time PIV is achieved using a laser sheet in the symmetry plane. This allows to detect the reversed flow region (striped area).

Here, u is the streamwise velocity component, H the Heaviside function and $\langle \cdot \rangle_T$ a time averaged value of its argument. Note that A_u is the time-averaged recirculation area for the uncontrolled flow. The cost function is aimed at minimizing the back-flow region and then the recirculation area, while penalizing the actuation cost:

$$J = \langle s \rangle_T + \gamma \langle |b| \rangle_T^2, \quad (14)$$

with b the control command (a signed value proportional to the flow rate through the jet) and γ a penalization coefficient. Although the choice for γ is arbitrary, the value represents a trade-off between the gain on the area reduction and the cost of actuation. Setting a low (respectively high) value of γ means that the performance of the system is much more (respectively less) important than the cost of the control. A balanced value can be derived by evaluating how much one is ready to spend in energy to achieve a given performance. The ratio between performance gain and actuation cost of the most effective open-loop control suggests a value close to $\gamma = 3/2$. The parameters used for GPC are summarized in Tab. V.

2. Results

The GPC process converges after 8 generations with 500 individuals each. Figure 17 shows the time series of natural and GPC controlled flow. The law found by GPC ensures a 58% reduction of the recirculation area, quite similarly to the best open-loop control at that Reynolds number. While the best open-loop case is an harmonic forcing around the vortex shedding frequency (1 Hz), a frequency analysis of the control signal for the GPC case

TABLE V. GPC parameters used for the control of the PMMH backward facing step flow.

Parameter	Value
n	500
P_r	0.1
P_m	0.20
P_c	0.70
n_p	7
n_e	1
Node functions $+$, $-$, \times , $/$, \exp , \log , \tanh	

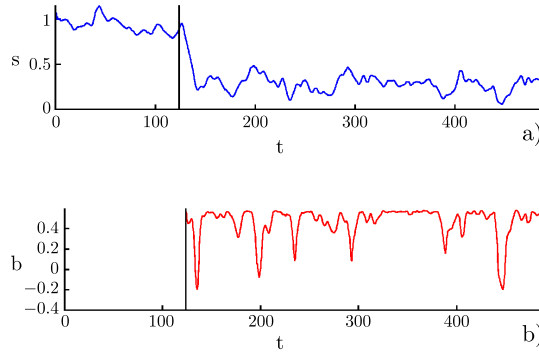


FIG. 17. Sensor signal s for natural and GPC controlled flow (a) and control signal b (b). The vertical black line shows when control starts. Time t is non-dimensionnallized using step height and free-stream velocity.

shows a dominant frequency around 0.1 Hz indicating that two different physical processes are used for the same reduction. Once again, shifting the operating conditions shows that the closed-loop control determined by GPC is much more robust than any open-loop forcing (Tab. VI): for both $Re_h = 900$ and 1800, GPC control manages to reduce the recirculation significantly (more than 50% while the same open-loop forcing is inefficient with less than 10% reduction out of the conditions where it has been designed). A detailed GPC study is described in [57].

TABLE VI. Performance of uncontrolled, best open-loop controlled and GPC controlled systems. While GPC performance is close to best open-loop control at the learning Reynolds number, the closed-loop control shines in maintaining performance in out of design conditions.

case	natural	best open-loop ($Re_h = 1350$)	GPC ($Re_h = 1350$)
J ($Re_h = 900$)	1	0.75	0.33
J ($Re_h = 1350$)	1	0.42	0.42
J ($Re_h = 1800$)	1	0.76	0.59

C. Wall turbulence control in LML experiment

GPC has been applied in the wall-turbulence wind tunnel at LML (Fig. 13c) with the goal to reattach the flow after the natural separation point occurring after a sharp ramp. The performances of the GPC have been compared to those of the best open-loop scheme obtained from an extensive parametric study.

1. Experimental setup

The LML wall-turbulence wind tunnel is a $1 \times 2 \text{ m}^2$ cross-section wind tunnel operated with a flow velocity up to 10 m.s^{-1} . The AVERT ramp model [58] is placed in the test section (Fig. 18b). This ramp was designed to select the pressure gradient of the upcoming boundary layer. A sharp edge is placed at the end of the ramp in order to force a separation of the flow at this fixed position. The actuation is made of 6 mm diameter angled jets (35° pitch angle, 125° skew angle) placed throughout the span, upstream of the separation in order to produce an optimal array of co-rotating streamwise vortices in the boundary layer [59]. These can be triggered in with an on/off mode using electro-valves with a maximal frequency of 300 Hz. Hot-films are placed in the descending part of the bump in order to record the wall friction at that position. These are used as sensors for the GPC process. Three hot-films are selected thanks to their sensitivity during previous open-loop and closed-loop attempts. We define the sensors signals as:

$$s_i = \frac{h_i - h_{i,u}}{h_{i,\max} - h_{i,u}}, \quad \text{with } i = A, B, C, \quad (15)$$

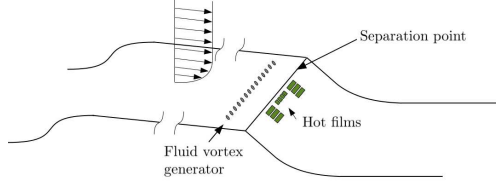


FIG. 18. Experimental configuration of the separating boundary layer in the LML wind tunnel. An array of co-rotating fluid vortex generators is placed upstream of the separation. The hot-film rake allows to qualify the friction at the wall, indicating potential re-attachment.

TABLE VII. GPC parameters used for the control of the LML separating boundary layer

Parameter	Value
n	500
P_r	0.1
P_m	0.25
P_c	0.65
n_p	7
n_e	1
Node functions $+$, $-$, \times , $/$, \exp , \log , \tanh	

where h_i is the raw voltage output of sensor i , $h_{i,u}$ the average voltage for the uncontrolled case (corresponding to a separated flow and low friction) and $h_{i,\max}$ the average voltage for the most efficient forcing, leading to maximal friction, in the case of constant blowing. As can be seen, the effectiveness of the actuation in reattaching the flow is here characterized by maximizing the wall friction (it was verified in previous experiments that the sign of the wall friction does not play a role [59]). The goal function used in this experiment is aimed at maximizing the friction while penalizing the actuation cost:

$$J = \left(\frac{1}{3} \sum_{i=A,B,C} \langle s_i \rangle_T^2 \right)^{-1} + \gamma \langle b \rangle_T^2, \quad (16)$$

with b the actuation value (0 or 1) and $\gamma = 2$ a penalization coefficient. Sensors A, B and C are hot-films placed the closest to the separation line at different span-wise position. Finally a PIV plane is setup in the symmetry plane to assess the effectiveness of the reattachment [58]. The parameters used for GPC are summarized in Tab. VII.

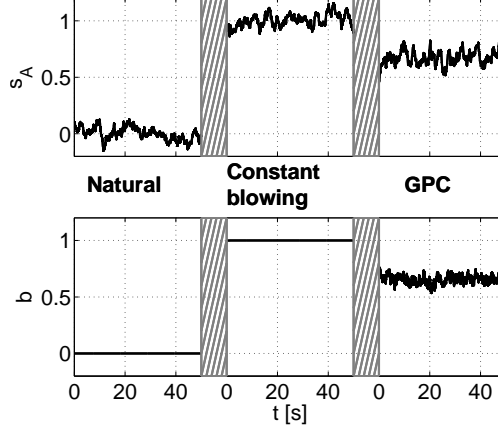


FIG. 19. Time series of sensor A (top) and low-pass filtered control signal (bottom) for uncontrolled (left), open-loop constant blowing control (center) and GPC control (right).

2. Results

Natural, constantly forced and GPC controlled time series of the A sensor and filtered actuation are displayed in Fig. 19. According to (16) and (15), the J value for the natural case is infinite as $\langle h_i \rangle_T = h_{i,u}$ in this case. We observe a 33% reduction of J in the GPC case compared to the constant blowing actuation (Tab. VIII). Nevertheless Fig. 19 indicates that the friction is lower in the GPC case than in the constant blowing case (though the reattachment is effective, see Fig. 20). All previous studies have shown that at the selected operating conditions, the influence of the frequency in the actuation is limited. The parametric study with respect to the duty cycle of an harmonic forcing reveals that above a certain frequency the relation between duty cycle and friction is monotonic with a positive slope. In few words, the more you blow, the more you re-attach. Previous control attempts revealed that the constant blowing is actually the best control configuration with respect to wall friction [59]. By penalizing the actuation strength in the control objective function, we actually select an operating point (which is reproducibly reached by several successive attempts) while applying the GPC process. All the control laws selected by GPC exhibit the same behavior: a rather high frequency content in the actuation signal with an average duty cycle around 70%. Different parameters and conditions have been tried and will be summed-up in an upcoming communication.

TABLE VIII. Respective J values for uncontrolled, best open-loop constant blowing control and GPC control. The uncontrolled J value is theoretically infinite and the constant blowing has its J value fixed by construction at 3. GPC achieves a 33% reduction of the J value.

case	natural	best open-loop	GPC
J	∞	3	2.1

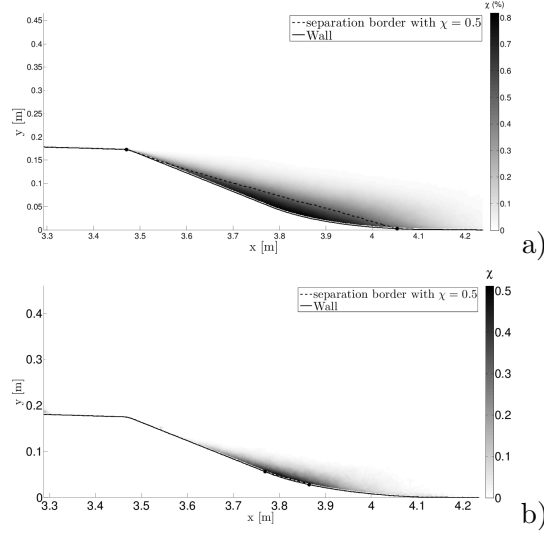


FIG. 20. Cartographies of back-flow coefficient χ , the ratio of the negative over positive value for the streamwise velocity at the considered point [60]. a: Uncontrolled flow. b: GPC controlled flow. The separation has been drastically reduced. For both cases the iso-line at $\chi = 50\%$ has been traced.

D. Wall turbulence control in PRISME experiment

GPC has been applied in the PRISME wall-bounded turbulence experiment with the goal to reattach the flow after the natural separation point occurring downstream a sharp ramp. A picture of the experimental setup is given in Fig. 13d. The design of PRISME experiment is close to that of the LML described in the previous subsection. Both experiments are involved in the SepaCoDe project funded by the French National Research Agency (ANR). These experiments differ by the oncoming boundary layer relative to the ramp height, the operating free-stream velocity, the physical time-scales and the properties of the unsteady vortex generators. The performances of the GPC have been compared to those of the best open-loop scheme obtained from an extensive parametric study.

1. Experimental setup

The experiments have been carried out in the Malavard closed-loop wind tunnel at the PRISME laboratory [61]. The square test-section is 2 m wide and 5 m long. A settling chamber and a strong contraction located upstream the test section ensure a residual turbulence intensity as low as 0.4%. A model is installed at the middle height of test section and it spans

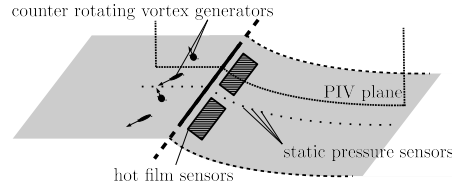


FIG. 21. Experimental configuration of the separating boundary layer in the PRISME wind tunnel. The jets are placed to generate counter-rotating streamwise vortices. Hot-film sensors are placed after the separation and static pressure sensors are located in the symmetry plane.

the entire width of the test section. A massive separation is produced by a backward-facing ramp whose height and length are $h = 100$ mm and $\ell = 470$ mm, respectively. The onset of the separation is fixed by a sharp edge with a 25° slant angle. The free-stream velocity U_∞ is 20m.s^{-1} yielding a ramp-height-based Reynolds number $Re_h = U_\infty h / \nu$ of around 1.3×10^5 .

For control purpose, 54 counter-rotating unsteady vortex generators (UVG) have been implemented one boundary layer thickness upstream of the sharp edge ramp (see Fig. 21). Their design and location have been extrapolated from the results reported by [62–64]. The jet velocity ratio is $V_{jet}/U_\infty = 3$. Each actuator is composed of two counter-rotating jets each with hole diameter of $\phi = 1.2$ mm. They present a pitch angle of $\alpha = 135^\circ$ are skewed by $\beta = 45^\circ$ and have a distance of $\lambda/\phi = 15$ between them. The distance between the center line of two consecutive VGA is $L/\phi = 30$.

Similarly to section IV C, temporal variations of wall-shear-stress are recorded by means of two hot-films located on the ramp together with the jet actuation collected from a mass flow controller. Furthermore, the pressure distribution along the model is acquired in order to compute the global properties of the flow. A PIV equipment is also used in order to record the recirculation length (see Fig. 21).

The sensors s_i used for the GPC control law are based on the hot-film signals:

$$s_i = \frac{h_i - h_{i,u}}{h_{i,\max} - h_{i,u}}, \text{ with } i = A, B, \quad (17)$$

where h_i is the raw voltage output of sensor i , $h_{i,u}$ the average voltage for the uncontrolled case (corresponding to a separated flow and low friction) and $h_{i,\max}$ the average voltage for the most efficient forcing, leading to maximal friction, in the case of constant blowing.

The evaluation of the individual is achieved according to the following cost function:

$$J = J_{\text{HF}} + \gamma_{\text{pstat}} J_{\text{pstat}} + \gamma_{\text{act}} J_{\text{act}}, \quad (18)$$

with J_{HF} being an evaluation of the friction recorded from the hot-films, J_{pstat} an evaluation based on the static pressure distribution and J_{act} an evaluation of the actuation cost. $\gamma_{\text{pstat}} = 1/200$ and $\gamma_{\text{act}} = 0.6$ stand for penalization coefficients. The evaluation based on the friction is defined as:

$$J_{\text{HF}} = \frac{1}{N_{\text{HF}}} \sum_{i=1}^{N_{\text{HF}}} \left[1 - \tanh \left(\frac{\langle \text{HF}_i \rangle_T}{\langle \text{HF}_{i,0} \rangle_T} - 1 \right) \right], \quad (19)$$

where $N_{\text{HF}} = 2$ is the number of hot-film sensors, HF_i the sensor value collected from the i^{th} hot-film and HF_0 the hot-film sensor value when no actuation is present. The value of J_{HF} is 1 when no effect is recorded and approaches 0 as the friction increases. The evaluation based on the static pressure is defined as:

$$J_{\text{pstat}} = \left\langle \frac{1}{0.1 + \sum_i (p(x_i) - p_u(x_i))^2 \frac{x_{\max} - x_i}{x_{\max} - x|_{x=0}}} \right\rangle_T, \quad (20)$$

with x_i the position of the i^{th} pressure tap after the edge, $x|_{x=0}$ the position of the pressure tap closest to the edge, x_{\max} the furthest downstream pressure tap, $p(x_i)$ the static pressure recorded at position x_i and $p_u(x_i)$ the static pressure recorded at position x_i in the uncontrolled case. $\langle \cdot \rangle_T$ is the average over the evaluation time T . J_{pstat} is equal to 10 when both controlled and uncontrolled pressure distributions collapse and goes to zero when they deviate from each other, with a linearly increasing weight when approaching the separation point. The evaluation of the actuation cost is defined as:

$$J_{\text{act}} = \left\langle \frac{Q}{Q_u} \right\rangle_T, \quad (21)$$

where Q is the flow-rate and Q_u the flow-rate under constant blowing. J_{act} is equal to 1 for constant blowing and is null when no actuation is recorded. The parameters used for GPC are summarized in Tab. IX.

TABLE IX. GPC parameters used for the control of the PRISME separating boundary layer

Parameter	Value
n	100
P_r	0.1
P_m	0.2
P_c	0.7
n_p	7
n_e	1
Node functions +, −, ×, /, exp, log, tanh	

TABLE X. Cost function values, separation and actuation properties for the natural, best open-loop and GPC cases.

Case	natural	open-loop	GPC
J	50.4	0.291	0.32
L_{sep}/h	5.4	3.14	3.16
$c_\mu (\times 10^{-4})$	-	16.51	13.66

2. Results

The pressure distribution obtained in controlled cases is compared to that of the baseline flow in Fig. 22(a). Both open-loop and GPC schemes lead to a reduction of the mean recirculation region since the recovery region associated to the pressure plateau is shifted upstream. Noticeable is the acceleration of the flow induced by the UVGs upstream the sharp edge location ($x/h = 0$) as emphasized by the strong decrease in pressure. However, pressure distributions computed for the best open-loop and GPC almost collapse meaning that from a global viewpoint, the efficiency of both control approaches are mostly equivalent. This is confirmed by the measurement of the separation length L_{sep} from the PIV dataset which is reduced by about 40% when control is applied (see Tab. X). Nevertheless, the actuation cost to achieve the same separation reduction is significantly lower ($\approx 20\%$) for the GPC as evidenced by the momentum coefficients c_μ ($\sim \frac{S_j d_c V_j^2}{1/2 S_{ref} U_\infty^2}$ with S_j the total cross section of the jets, d_c the duty cycle and S_{ref} the surface of the ramp) reported in Tab. X.

One of the main differences of the actuation between our best open-loop case and the GPC relates on the frequency distribution (computed from a zero-crossing algorithm applied on the mass flow controller signal) of the blowing (see Fig. 22(b)). Unlike open-loop control for which the blowing frequency is unique (here 30 Hz), the frequency distribution of the GPC is broad-band. More surprisingly, the frequency range of the GPC does not match that of the best open-loop case meaning that even though both control approaches lead to the same separation reduction, their underlying mechanisms differs significantly. A more detailed study of the flow physics will be reported in an upcoming publication.

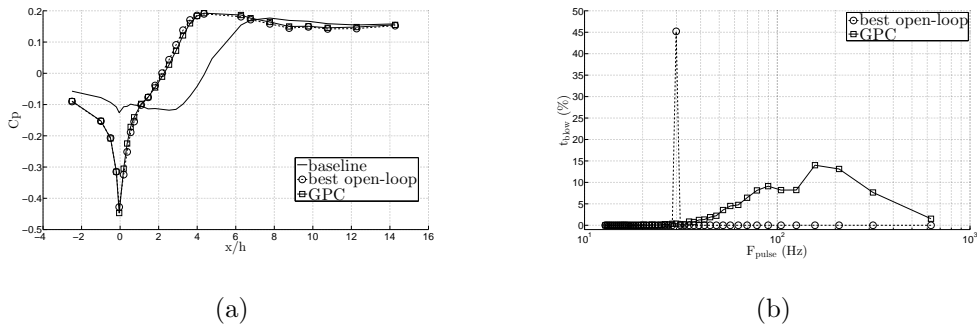


FIG. 22. (a) Pressure distribution along the ramp. (b) Frequency distribution of the blowing.

V. CONCLUSIONS

We propose a model-free optimization of sensor-based control laws for general multiple-input multiple-output (MIMO) plants, the 'Genetic Programming Control' (GPC). This model-free strategy, based on genetic programming, is aimed at offering a solution for the in-time closed-loop control of complex non-linear systems like turbulent flows. GP is one of the most versatile methods for function optimization in machine learning. GPC relies on the evolution of an ensemble (called 'generation') of general nonlinear functions (called 'individuals') and invests in exploring the function-space. Since GPC is an evolutionary algorithm, it has a large chance to detect and exploit otherwise invisible local extrema. In contrast, model-free adaptive control is particularly suited for adjusting one or few parameters of prescribed open- or closed-loop control laws to changing flow conditions. Such on-line parameter adaptation is not part of the presented GPC method but could — in principle — be included.

As our first test-case, GPC has been successfully applied to a closed-loop stabilization of a oscillator model detecting and exploiting frequency cross-talk in an unsupervised manner. Frequency cross-talk is of crucial importance for large-scale turbulence control with complex interactions between the coherent structures at different dominant frequencies: the mean flow changes on large time scales while the cascade to small-scale structures has small associated time scales. By definition, frequency cross-talk is ignored in any linearized system. Another successful demonstration of GPC is closed-loop control for the maximization of the Lyapunov exponent (stretching) of the forced Lorenz equations. Again, this increase of unpredictability is a highly nonlinear phenomenon.

Most importantly, GPC has also been applied successfully to a panel of experiments representative of the challenges encountered in modern turbulent fluid dynamics. In all cases GPC has been able to derive an effective control law in a model-free approach and with performances consistently beating the optimized comparison control studies. All of these control laws are closed-loop control laws which exhibit an inherent robustness when compared with open-loop forcing which is by far the most commonly used strategy for the control of turbulent flows. Furthermore GPC has sometimes derived control laws that exploit an unexpected mechanism as in the PMMH experiment on the backward facing step. GPC has proved to be effective in exploring the search-space characterized for each control problem by its cost function.

While it can be claimed that having to evaluate all individuals from each generation is a major time investment, the total time investment is not larger than with other methods, and is in fact mostly lower. First of all the approach is model free, which means that no prior experiments are needed, neither to perform a system identification or corroborate the model adequacy to the plant. GPC can be applied on any closed-loop ready system with very few preparations consisting mainly in implementing a function that can translate the individual and creating the communications to exchange the individuals and their cost values. With the accumulated experience, this is now a few hours task. Then, in most unfavorable case, i.e. in the PMMH water tunnel where evaluation time has to be larger due to the small velocities when compared to air flows, and where 500 individuals per generation have been employed, a full week of experiment has been employed. This means that only a week was needed to derive an effective in time closed-loop control law for an experimental flow. To our knowledge this beats any other approach. In wind tunnels, with the velocities involved

in the considered studies (of the order of 10 m.s^{-1}) and while dealing with small generations (50 individuals), a control law can be obtained within around 8 hours of operation.

While the optimality of the results from evolutionary algorithms can never be proved, the goal of the method never was to obtain the best possible control law, but obtaining one which is performing well within a range of the cost function values decided by the operator. GPC shines when no modeling of the flow can be obtained in finite time, thus obtaining a performing control law is already a major success.

GPC has overcome important technical challenges for in-time control: (1) the sensors show broadband frequency dynamics, (2) there is a large convective time delay from actuators to sensors and (3) the responses were found to be strongly nonlinear.

Summarizing, the model-free formulation of GPC gives rise to a high flexibility: it can be applied to any MIMO plant and use any cost function. Though a model is not needed, the more we know about the system, the better we can design the cost function according to the underlying physics and the better we can bias the control law selection. Further improvements can be expected from including actuation or sensor histories, like in ARMAX models [5]. The relation of tree depth, number of generations, number of individuals with convergence is subject of ongoing research and may boost the performance considerably.

The model-free control design is particularly interesting for experimental applications for which a model might not even be known, like for the control of some multi-phase or multi-physics flows with several phases, combustion or unknown non-Newtonian fluids. The following step in machine learning control will certainly be to extract knowledge from the results of GPC in order to build models for the flows by studying the set of best control laws obtained. This way the model-free GPC will play a major role in determining non-linear models for complex flows. We conjecture that GPC will play a similar role as control theory in the closed-loop control of turbulence and other complex flows.

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